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Robust Design Simulation: A Probabilistic Approach to Multidisciplinary Design

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Most current paradigms in multidisciplinary design analysis and optimization fail to address the presence of uncertainty at numerous levels of the design hierarchy and over the design process time line. Consequently, the issue of robustness of the design is neglected. An approach for the determination of robust design solutions is outlined in this paper, where uncertainty is quantified and its effects mitigated. The robust solution is found through maximization of the probability of an overall figure of merit achieving or exceeding a specified target. The proposed methodology is referred to as robust design simulation (RDS). Arguments as to why a probabilistic approach to aircraft design is preferable over the traditional deterministic approaches are presented, along with a step-by-step description of how one could implement the RDS. An application involving the high-speed civil transport is conducted as a case study to demonstrate the proposed method and to introduce an evaluation criterion that guarantees the highest customer satisfaction.

Introduction

THE design activity is concerned with a designer using available information to make intelligent decisions leading to optimal solutions.[1] The process of complex systems design (a collection of these activities over time) is inherently hierarchical. For example, the design of an aircraft usually involves the simultaneous design of its major systems, e.g., wing; its subsystems, e.g., control surfaces; and its individual components, e.g., actuators. The decision-making modifier, intelligent, implies the presence of accurate models of the system in question, its inputs, and its possible operating environment, as well as models of the uncertainties associated with those systems, inputs, and environment. Uncertainty, for now, is defined as the error between a mathematical model and reality, arising mainly as a result of a lack of knowledge available for constructing the model. In the aerospace disciplines of control-system design and structural design (particularly reliability analysis), the necessity of such models has long been recognized, and their use for decision making is part of the state-of-the-art in those particular fields. To a large extent, this same recognition has not extended to the field of aerospace vehicle synthesis and multidisciplinary analysis and multidisciplinary design optimization (MDA/MDO), particularly with regard to uncertainty. Research in MDA/MDO has been quite successful in modeling the data exchange involved in two and three discipline problems with a system-level objective.[2,3] But the complexity of the multidisciplinary environment in which vehicle design decisions must be made makes formulating and implementing the required uncertainty models difficult. Thus, a methodology for system synthesis and design is needed that is both multidisciplinary and able to deal with uncertainty and risk.

Research in areas such as MDA/MDO is critical for the successful, efficient design of future complex aerospace systems, particularly as historical databases (once the centerpiece of conceptual design) become increasingly obsolete. This database obsolescence began when product solutions departed from traditional designs (configurations outside historical databases, multirole missions, functionalities, etc.), and was then exacerbated by the inadequacy of process models (manufacturing methods, knowledge creation/exchange, design process management, etc.), as well. This dual-caused obsolescence highlights the often-neglected relationship between product and process concerns. A response to this situation is the integrated product and process development (IPPD) design paradigm.[4] The interrelation of product and process characterizations forms the heart of IPPD, and extends the scope of MDA/MDO. While in the past, vehicle MDO focused primarily on product concerns, IPPD specifically emphasizes simultaneous product, e.g., performance, geometry, and process, e.g., manufacturing, production, and market economics, recomposition. The primary motivation behind IPPD is an increased focus on development cycle time reduction and on design for affordability, defined as the ratio of a system's benefits over the cost of achieving those benefits. Addressing affordability in conceptual design shifts the fundamental question from Can the vehicle be built? to Should the vehicle be built? Progress made in the MDA/MDO field over the last 10 years has been hindered by these factors, most notably the failure to understand what is to be optimized and when. Key objectives (such as affordability) and key observations (such as the nondeterministic nature of the MDO problem and the importance of the modeling the human designer's decision-making activities), have been largely neglected.

Uncertainty permeates engineering analysis, particularly for approximations of high-fidelity analysis typically used in early design stages. Even the most elegant decomposition, approximation, and optimization schemes (the current focus of MDO according to Ref. 5) cannot properly account for imprecise contributing analyses, uncertain operating conditions, and ambiguous design requirements. The design approach developed in this paper builds on key developments in MDA/MDO, but takes the critical extra steps of addressing robustness at the system level through direct uncertainty representation at multiple levels of the design hierarchy. The construction of this paper consists of first establishing the need for robust vehicle design, next reviewing existing techniques found in the literature, and then presenting and demonstrating an innovative approach that extends the capabilities of those reviewed in

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finding robust solutions within a set of feasible and viable alternatives.

Robustness and Robust Design

Conceptual vehicle design involves determining an extremal value for a figure of merit, for that class of vehicles, as a function of selected free parameters (called design or control parameters). For example, one might search for the external geometry of a transport aircraft that minimizes the gross weight for a given mission. Sensitivities of gross weight to geometry parameters are used to direct the search for the optimum region of the design space. This approach is deterministic in that it assumes that there is no uncertainty (and thus no variability) in computing the objective and constraint function values. An alternative approach exists that is called robust design. During robust design, a designer seeks to determine the control parameter settings that produce desirable values of the objective function mean, while at the same time minimize the variance of the objective function probability density function (PDF).[6] Robust design, then, is a multiobjective and nondeterministic approach, and is concerned with both the objective function mean and the variability that result from uncertainty (represented through noise variables). Thus, uncertainty identification and modeling are prerequisites to performing robust design. In this setting, sensitivity analysis is concerned with both the mean and variance of the objective. The focus of this paper is not to investigate the variety of sources of uncertainty in aircraft design (though this too is important), but instead to understand the best way to proceed once uncertainty is identified. Deterministic optimization (the traditional approach in air vehicle design) simply neglects these uncertainties by assigning such quantities an assumed value, or ignoring their effects altogether. This practice is increasingly inappropriate in the development of affordable systems, where cost prediction and risk mitigation are of equal concern as vehicle performance.

A graphical representation of the concept of achieving a robust design is shown in Fig. 1, in terms of an L/D distribution over a range of lift coefficient (C_L) for two different hypothetical designs. For the same optimal C_L value, design 1 yields a higher maximum L/D. However, because the aircraft will inevitably be flown at off-design conditions, C_L will vary over the assumed operating range. Thus, each design has an associated variation of L/D. As depicted in Fig. 1, this variation depends on the setting of design variables, causing design 1 to yield a larger variation in L/D than design 2. If the objective is to create a robust design, then design 2 may become more desirable, because it yields a higher probability of achieving L/D values greater than a specified minimum during operation. In effect, the better design-point performance is traded for the superior off-design performance. The ability to capture tradeoffs such as these associated with robustness is at the heart of the robust design simulation (RDS) framework proposed in this paper.

While the concept of robustness in product and process design is intuitively sensible, implementations and realizations of it for aerospace systems design are few. General formulations have been developed, however, in other fields. Perhaps the most well known is the Taguchi parameter design approach.[6]

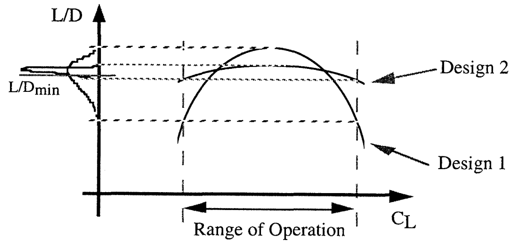

Fig. 1 Change in sensitivity of L/D for two hypothetical designs.

This approach originated as a quality-enhancement mechanism in the area of industrial systems and manufacturing. It is characterized by a separation of control and noise variables into different arrays, in preparation for statistical experimentation to determine the value of an objective. After the assigned number of experiments (or simulations) are executed, a signal-to-noise ratio (SNR) is computed that measures the product performance and its variation as a function of control variable settings. It is a nonprobabilistic approach in that there is no uncertainty modeling through probability distributions. In recent times, several drawbacks of Taguchi's robust parameter design approach, of both a practical and theoretical basis, have been cited. Myers et al.[7] and Tsui[8] are two excellent references that detail these identified shortcomings as follows: a model of the response is not formed (preoccupation with optimization), interactions among control or noise variables are not modeled, array designs are uneconomical compared with fractional factorial designs, and the SNR is insufficient as a figure of merit. Despite these limitations, some useful applications of Taguchi's approach for multidisciplinary problems in aerospace have been conducted, notably those by Yurkovich.[9]

More recent approaches focus on the use of combined arrays. This means control and noise variables are grouped together into one response model, which is subsequently used for robust design purposes. This formulation usually utilizes the response surface methodology (RSM)[10] to form this combined model in the form of response surface equations (RSEs). The RSM will be described later in this paper as an integral part of the RDS method. The power of employing the combined array approach in a robust design setting for aerospace problems was first demonstrated by researchers at the Georgia Institute of Technology.[11,12] In Ref. 11, the authors promote the position of openness and robustness in a decision-based engineering design framework, where robustness is approached in terms of minimizing the gradient of the objective with respect to deterministic design variables. The combined array response model is used in a formal design space search for optimal solutions, or regions of good solutions, based on some measure of robustness. Along these lines, Chen et al.[13] uses a robust concept exploration method, to find robust solutions based on models of the response mean and variance. A limitation of this approach is that the noise variables are assumed independent and the noise variability modeling is restrictive (as in Taguchi's approach).

The RDS method described in this paper is a combined array approach with additional characteristics that provide enhanced information for use in finding robust designs while overcoming some of the deficiencies in the previously described methods. RDS combines the response model with a Monte Carlo simulation to construct cumulative distribution functions (CDF) and PDFs for the objective and constraints. These functions are used to guide the designer in making decisions and trades at the system level. It is the only truly probabilistic approach among those discussed. Under development over the last three years,[12,14–16] the approach treats uncertainty directly and uses the CDFs and PDFs of the objective as the decision functions in robust design. As pointed out in Ref. 17, there is a concern that the response surface approximations may not generate accurate sensitivities that are required for robust design. This important observation is addressed here and must be addressed in any use of the combined array approach.

A demanding application has been chosen to illustrate portions of the proposed RDS method: the conceptual synthesis and selection of a high-speed civil transport (HSCT). Because this HSCT must be economically competitive with current long-range subsonic transports, its design must be focused on estimates of its economic viability, or more accurately, the probability of its economic viability in the presence of a variety of uncertainties.

RDS for Aircraft and the Probabilistic Approach

The ultimate goal of RDS is to aid a designer in improving system affordability, over a range of possible operating conditions, by choosing designs from within a set of feasible alternatives. Unique to aircraft design, an aircraft synthesis and sizing process, utilizing appropriate analytical tools, evaluates the system value to the customer, i.e., passenger, and subsequently, airline, for each aircraft configuration through selected objectives such as performance, cost, profit, reliability, or some all-encompassing evaluation criterion. Regardless of the defined objective, customer satisfaction can only be achieved if all system design and environmental constraints are met. This algorithm is displayed in Fig. 2, depicting the dependence of the objective on economic and discipline uncertainties as well as performance and schedule risk for technology infusion.

The core of the RDS consists of vehicle sizing and synthesis combined with an environment to simulate its operation. A synthesis and sizing tool is, by definition, multidisciplinary and hierarchical. The top level of the hierarchy is the system level, where the responses of interest such as weight, cost, and performance are computed from a mission analysis. The second level consists of the contributing disciplines such as aerodynamics (in the form of drag polars), propulsion (in the form of thrust tables, fuel flow), and structures (in the form of component weights) as functions of elementary variables. Some of these variables may be common among several disciplines. The RSM introduced earlier can be applied at both levels of the hierarchy. The application of RSM in the discipline level provides the required coordinating mechanism between the discipline and system levels, whereas application at the system level itself provides the ability to perform robust design searches. In this paper, the focus is on exemplifying the system level application. References 16, 18, and 19 discuss the discipline level application of RSM to tackle multilevel, multidiscipline problems. As a final note, discipline-level RSEs can be used to capture the effects of technology integration and then be rolled into the system-level metrics.[20]

Returning to Fig. 2, feeding the core synthesis capability are customer requirements, physics-based models for disciplinary analyses, uncertainties, and a suite of candidate new technologies that may be required to make the system feasible. The objective that drives the decisions made in this environment is customer satisfaction measured through affordability.

The procedure for conducting this IPPD approach employs the use of a design of experiments (DoE) to facilitate the development of the aforementioned RSEs, which approximate sophisticated, computationally intense disciplinary analyses tools with second- (or higher) order polynomial equations. The use of RSEs is an integral part of RDS, calculated using a combined array approach.

Uncertainty and risk associated with a system can be modeled as random variables with probability distributions assigned for each variable inherent to the system. These random variables introduce an undesired variability in the objective, which is also modeled as a probability distribution. While some other robust design methodologies emphasize optimization of some weighted combination of objective mean and variance,[11,17] RDS assumes that maximum customer satisfaction is achieved by maximizing the probability, P, of achieving a target value for the customer's objective function, Y. In addition, the method relates this probability qualitatively and quantitatively back to design and control factors of the system. Hence, by maximizing the evaluation criterion, the probability of achieving objective function values that are smaller (or greater) than a desired target value, $P(Y < T)$, or $P(Y > T)$, a design can be found that guarantees the highest customer satisfaction while satisfying imposed constraints.

The implementation of this methodology is illustrated in Fig. 3 and the following five steps:

Step 1

After a complete list of pertinent design variables and their ranges of validity are established, a designer performs a screening process to determine the most important subset from the complete list of parameters that describe the system. As a general approach in the RSM, if the number of design variables in the model is above 10, a screening is first conducted before the actual model building experiment is run. This screening test typically uses a two-level fractional factorial DoE to test a linear model, and thus, estimates only the main effects of the design variables on the response.[10] However, it allows for the investigation of a high number of variables to gain an initial understanding of the problem and design space. To identify the variables with relevance to the objective function in a Pareto plot (Fig. 4) is generated. It is used by a

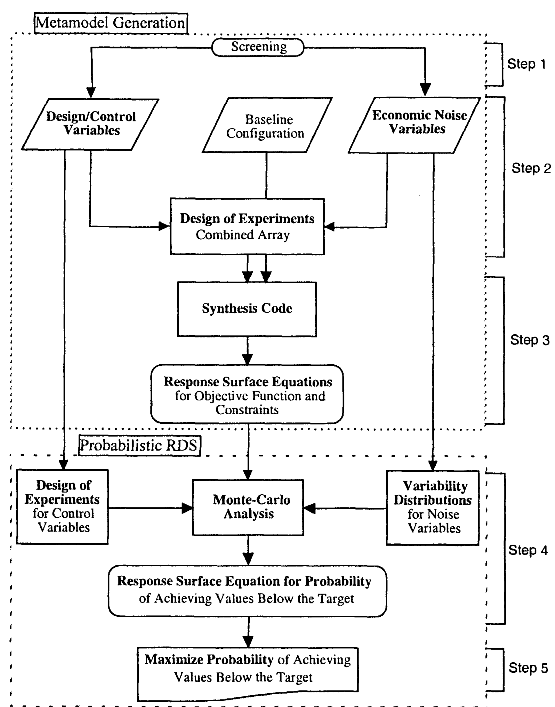

Fig. 3 RDS methodology flowchart.

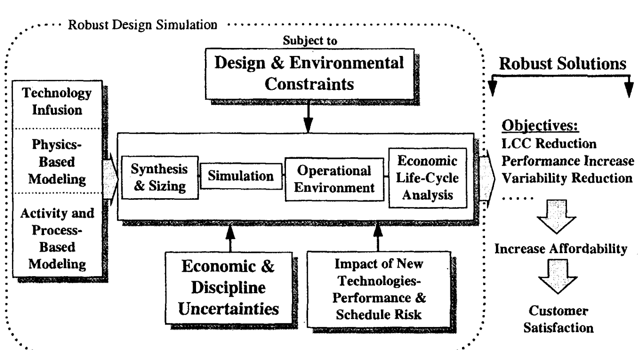

Fig. 2 Overview: RDS.

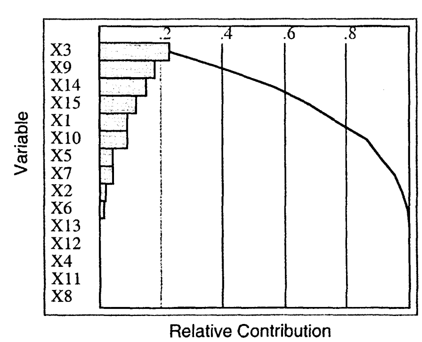

Fig. 4 Notional Pareto plot.

designer to visualize the importance of each variable to the objective function model by a bar, with the length of the bar corresponding to its importance. In addition, the solid line in Fig. 4 represents the cumulative effect of all normalized variables. The example that is shown employed 15 generic variables. It is seen that variables X3, X9, X14, X15, X1, and X10 have high significance, X5 and X7 have modest significance, and X2 and X6 are somewhat significant to the response. Depending on the model size desired in the next step, X2 and X6 can be excluded from the model, because their influence is likely to disappear in the noise of the problem. X13, X12, X4, X11, and X8 clearly have very little effect on the response (within this bounded design space), and can therefore be screened out. Reference 10 contains a more detailed description of the generation process for the Pareto plot.

Steps 2 and 3

The goal of these steps is to use the RSM to obtain a set of RSEs for the system level objective and constraints as a function of control and noise design variables. Once the screening is complete (step 1), a new DoE is created using only the selected variables. Typically, this experimental design allows for an estimation of main effects, interactions, and quadratic effects of the selected set of variables. For each individual experiment required, an analysis code is executed at the appropriate input levels, and the responses of interest are recorded. The use of DoE/RSM in the setting of multidisciplinary aircraft design to generate RSEs has been exercised in several studies.[14–16,21–23] The RSEs represent a smooth approximation of the behavior of the code in predicting the responses. However, they are only valid within the original ranges of the variables used in the DoE, i.e., extrapolation outside the modeled design space is not recommended. These equations are an integral part of the RDS methodology because they enable the use of a Monte Carlo simulation to determine the probabilistic characteristics of the objective. This is discussed next.

Step 4

In step 3, an RSE is established that links the objective function, to discipline level variables. The goal of step 4 is to create an RSE that predicts the probability of achieving (or surpassing) a target value for the system objective function. To identify a robust solution, one must identify those control variables that minimize the influence of noise variables, and shift the mean of the objective towards the target. In other words, the interaction between control and noise variables must be identified; a task that is particularly easy in the setting of a second-order RSE and an improvement over the Taguchi approach (which does not quantify explicitly these control/noise interactions). An example is provided in Fig. 5, where the control variable X1 is considered with respect to a noise variable and objective function, which is to be minimized. Figure 5a indicates that a high setting of control-variable X1 results in low variability of objective, which is desirable. However, Fig. 5b indicates that a high setting for X1 increases the value of the objective, an undesirable situation. To obtain a robust solution, some trade of mean performance vs variability reduction must occur. RDS facilitates this tradeoff process.

A new design of experiments table is used for the control variables only. At the same time, a designer selects probability distributions for the noise variables, based on expected or past behavior, if known. These distributions can be, and usually are, nonnormal. For each case in the control-variable DoE, a Monte Carlo simulation is executed based on the noise variable. Because each Monte Carlo simulation requires at least 5000 function calls, one can see the importance of the rapidly evaluated RSE in place of the actual analysis code in the process. In the application problem presented later in this paper, the use of the actual analysis code would result in about 805 days of CPU time (estimated based on a dedicated IBM RS6000-320H) to complete step 4, as opposed to ~4.3 h using RSEs. In addition, the Monte Carlo simulation has been chosen over an analytical method for the distribution generation because it does not need the simplifying assumption of normal distributions for the noise variables required by most analytical methods. However, research is under way by the authors to avoid the main limitation resulting from the use of Monte Carlo for probabilistic analysis: the necessity of approximating functions, e.g., RSEs, for function evaluations. Preliminary results of this effort are found in Ref. 24. Each of the simulations generates a frequency plot for the objective, similar to the one in Fig. 6. Each resulting frequency plot is approximated by a standard continuous PDF (and associated CDF). This distribution fitting process, employing the chi-square ranking method,[25] yielded a gamma distribution as the best approximation for all cases of this study. The PDF (f) for the gamma distribution is displayed in Eq. (1), where the parameters location (L), scale (α), and shape (β) uniquely define each distribution:

$$f(x) = \frac{[(x - L)/\alpha]^{\beta-1} \cdot \exp\{-[(x - L)/\alpha]\}}{\alpha \cdot \Gamma(\beta)} \tag{1}$$

To end step 4, the resulting series of CDFs are used to determine the $P(Y \leq T)$ value for each case in the DoE, and an RSE for this probability as a function of control variables is formed. In other words, the resulting equation links the probability of achieving values smaller than a target to control variables, allowing a designer to complete a robust design process by selecting the control variables that maximize this probability of success.

Step 5

After the completion of step 4, the goal in step 5 is to obtain a robust solution by maximizing $P(Y \leq T)$, while satisfying all imposed design and environmental constraints. As shown in Fig. 7, a feasible but economically nonviable solution can generate an objective function distribution that has very little, or in this case, no probability, of achieving the target value, symbolized by the distribution on the right. To increase the probability of achieving the target value, the distribution should be shifted to the left. The distribution that yields the highest probability, on the left in Fig. 7, is called the optimal solution, because it maximizes the new probability while satisfying all imposed constraints. This ability to perform a constrained, probabilistic optimization is another advantage of the RDS method over, e.g., the Taguchi method.

However, the optimal solution may not satisfy a required minimal probability (usually 75–90%) in all cases. If so, several options are left to achieve the desired level of certainty of success: relaxing the stringent target requirement by introduc-

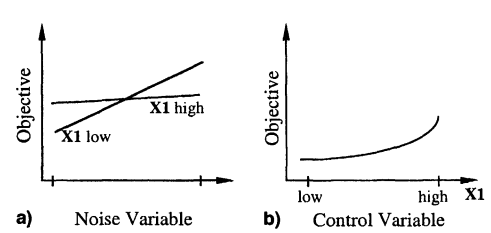

Fig. 5 Typical tradeoffs involved in robust design.

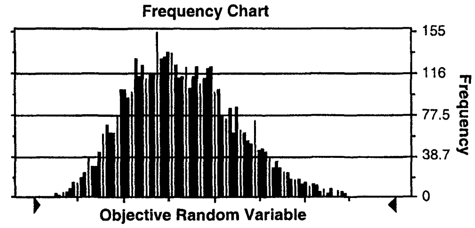

Fig. 6 Example frequency distribution for an objective.

ing a fare premium; changing some of the economic assumptions by guaranteeing a higher market share or schedule provisions; relaxing design constraints; changing the baseline by technology infusion; or, if everything else fails, accepting the solution at a lower rate of probability of achieving objective function values below the target; hence, taking a greater risk.

The robustness optimization in RDS is summarized as follows (assuming Z is to be minimized if this were a deterministic problem):

Maximize: $\text{Prob}[Z(\boldsymbol{X}, \boldsymbol{Y}) \leq z_0]$
Given: Z = overall objective (a measure of merit)
$\boldsymbol{X}$ = vector of i deterministic (control) variables (the free parameters)
$\boldsymbol{Y}$ = vector of j random variables, defined by individual uncertainty models
z_0 = target (a particular value of Z) supplied by the customer/decision maker
Satisfying: imposed performance constraints; side constraints on $\boldsymbol{X}$

RDS Case Study: A HSCT

The purpose of this case study is to exemplify the steps involved in executing the RDS, including details on the modeling, analysis, and general use of probabilities. Before developing the case study, the baseline vehicle under study is described.

Baseline Vehicle Description

The HSCT is envisioned to be an aircraft capable of cruising supersonically (~Mach 2.4), and carrying around 300 passengers to destinations up to 5500 n miles. These baseline requirements were determined from collaborations with NASA and industry over the last several years. Aside from the challenges of meeting these aggressive performance levels, stringent economic and environmental requirements are being placed on this aircraft. For example, the vehicle must abide by restrictive FAR Stage III noise regulations, be comparable in safety and comfort to the current long-range subsonic fleet,

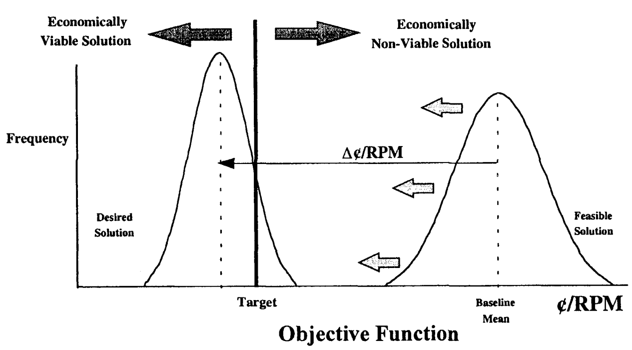

Fig. 7 Distribution shift required to maximize the probability of achieving a target.

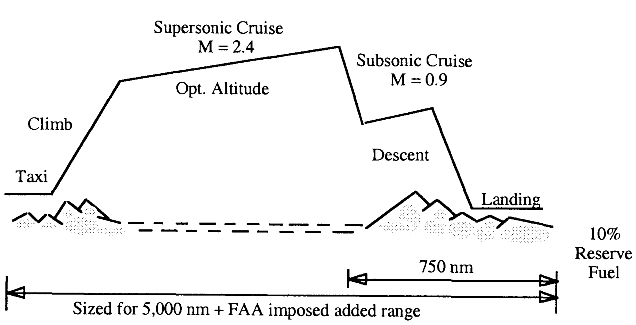

Fig. 8 Hypothetical HSCT baseline mission profile.

and provide economic benefits for both the airline and manufacturer while offering an affordable ticket price to the passenger. Affordability will be the ultimate metric that decides the fate of the vehicle. Hence, the objective function chosen for this study is the required average yield per revenue passenger mile ($/RPM), a widely accepted airline affordability metric. This metric captures the interest of the passenger (who desires a low fare), and of the airline (which desires a guaranteed return on investment).

The mission profile used to size the HSCT configuration is depicted in Fig. 8. To model the economics of the aircraft correctly, a distinction must be made between the economic and design ranges. The economic range represents the average distance an aircraft will fly from one airport to another during its life, whereas the latter is the maximum distance the vehicle is able to fly by design. The most important mission as well as geometric parameters of the baseline HSCT configuration are summarized in Table 1.

Maximizing the probability of success for a supersonic transport vehicle is a difficult task, as illustrated by the following conflicting design objectives. Choosing a wing planform shape, e.g., is driven by the need for efficient performance at both subsonic and supersonic cruise conditions, a conflicting design objective in itself.[16] Furthermore, the trades involved in planform selection are being complicated by different discipline considerations for aerodynamics, structures, propulsion, etc., and the presence of design and performance constraints at the system level that are directly related to the wing. The limit on approach speed, e.g., is mostly a function of wing loading. Fuel volume requirements impact the wing size and shape; both become sizing criteria and are treated as constraints that tend to increase the wing's size. On the other hand, increased wing area yields higher induced and skin friction drag, increasing fuel consumption. Additional design challenges are presented by takeoff and landing field length limitations (less than 11,000 ft) that are also modeled as design constraints for this study. Finally, a fuel volume ratio, *Rf*, can be tracked in the form of a ratio of available fuel volume to required volume to complete the mission. Hence, the *Rf* must be greater than one.

The objective, $/RPM, along with all imposed constraints considered for this study, are summarized in Table 2. All responses presented in this table are modeled by the sizing and synthesis tool chosen for this case study, the flight optimization system/aircraft life cycle cost analysis (FLOPS/ALCCA) code. The FLOPS, developed at the NASA Langley Research Center,

Table 1 Summary of the baseline HSCT

Parameter	Baseline
Cruise speed	Mach 2.4
Cruise altitude	~63,000 ft
Design range	5000 nm
Payload	300 passengers
Sustained load	2.5 g
Fuselage length	280 ft
Wingspan	155 ft
Inboard wing sweep	74 deg
Outboard sweep	45 deg
Wing reference area	9000 ft^2

Table 2 Summary of objective and constraints

Response	Requirement
$/RPM	Minimize
Gross weight	<1,000,000 lb
Approach speed	<154 kn
Fuel volume available/required	>1
Landing field length	<11,000 ft
Takeoff field length	<10,500 ft

is an aircraft-sizing code capable of analyzing a wide range of vehicles, including fighters, transports, and general aviation aircraft.[26] The ALCCA, developed at the NASA Ames Research Center, takes the component weights and fuel weight from a converged FLOPS sizing run as well as other parameters related to airline and manufacturer business practices, e.g., required return on investments.[27] The primary output of ALCCA used here is the $/RPM and acquisition cost. FLOPS and ALCCA have been integrated into one executable program. The control and noise variables are inputs to this combined sizing/economics program.

Step 1: Screening

The purpose of this step is for a designer to identify the subset of important variables to the problem. System-level screening tests for an HSCT have been described in previous papers by the authors.[15,16] Using these results, the variable subset after screening is shown in Table 3. The baseline aircraft, with some key variable definitions, is shown in Fig. 9. The selected set of variables at this stage includes control and noise/uncertainty variables. Noise variables can encompass such considerations as turbomachinery component efficiency, engine installation losses, and economic parameters such as fuel price and passenger load factor. Further, imprecision in underlying engineering analysis could also be modeled as uncertainty. For this particular case study, the assumption is made that all uncertainty is of an economic type, so that only economic responses (such as $/RPM and acquisition cost) will be probabilistic.

Table 3 Summary of control (C) and noise (N) variables

Name	Type	Variable	Range
Thrust/weight ratio	C	TWR	0.28–0.32
Wing area	C	S	8500–9500 ft^2
Longitudinal kink location (normalized by semispan)	C	x1	1.54–1.62
Spanwise kink location (normalized by semispan)	C	y1	0.5–0.58
Turbine inlet temperature	C	TIT	3000–3250°F
Fan pressure ratio	C	FPR	3.5–4.5
Fuel cost	N	$-fuel	0.55–1.1 $/gal
Load factor	N	LF	0.55–0.75
Economic range	N	Ec-range	3000–5000 nm

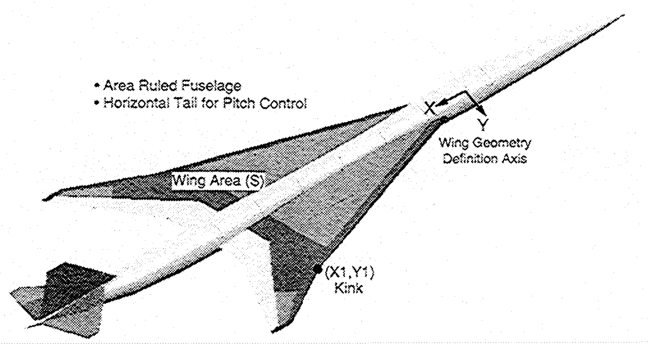

Fig. 9 HSCT baseline with geometry definitions.

The ranges for the selected variables that describe the design space under study are also listed in Table 3. Thrust-to-weight ratio, one of the main sizing variables, is the ratio of engine thrust over the gross weight of the vehicle. Wing reference area replaces wing loading as a sizing variable and contributes also as an aerodynamic variable. The kink location is found to be the most influential aerodynamic effect contributing to the $/RPM objective and the constraints.[16] Turbine inlet temperature (TIT) and fan pressure ratio (FPR) are the main engine cycle variables contributing to objective function and the constraints. Fuel cost ($-fuel), modeled over the life of the aircraft, includes the rather unlikely case of an oil crisis, yielding the relatively large variation. Load factor (LF) is strictly an economical factor, describing the ratio of passengers boarded on a given trip to available seats. The economic range is at least 3000 nm, and is most likely around 3200 nm. The selected set of variables clearly indicates the multidisciplinary nature of the problem because the HSCT is represented in the synthesis process by at least two variables each from the aerodynamics, sizing, propulsion, and economic disciplines.

Steps 2 and 3: RSE

The goal of these steps, once again, is for a designer to use RSM to obtain a set of RSEs for the system-level objective and constraints, as a function of the control and noise design variables in Table 3. To construct this equation, a face-centered central composite design (CCD) experimental design is employed. This requires 531 function calls to FLOPS/ALCCA, and a second-order polynomial is fitted to these obtained data. These RSEs are displayed graphically in Fig. 10 in the form of prediction profiles for each variable. Three values appear

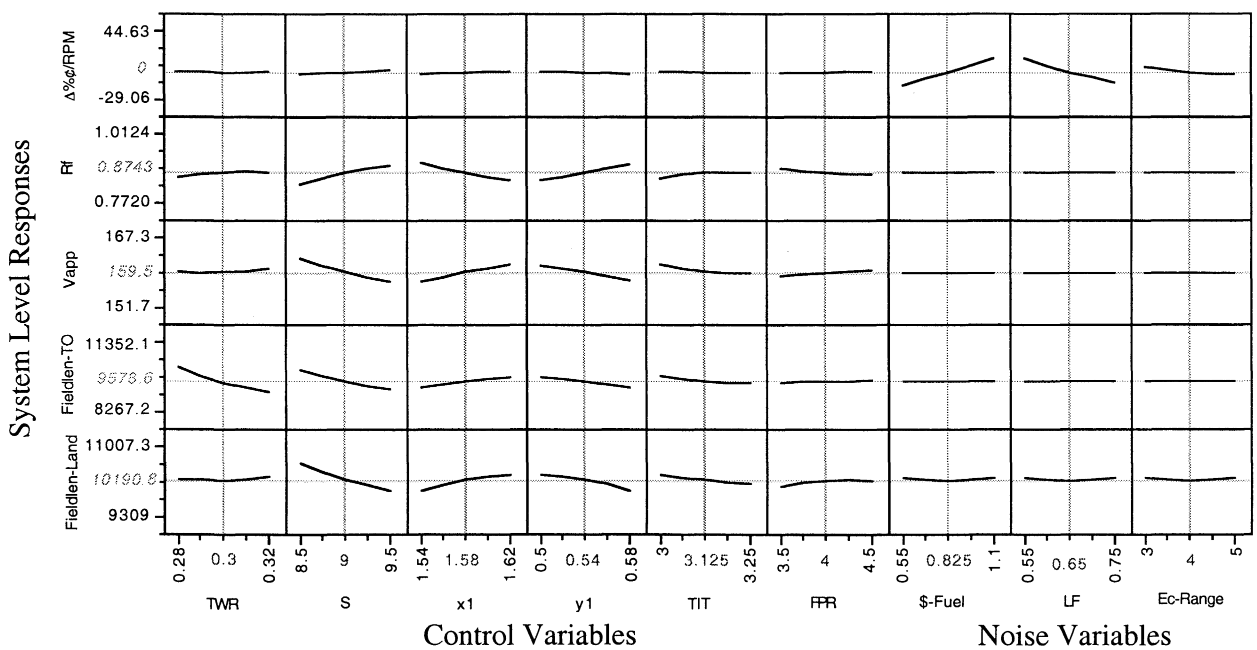

Fig. 10 Prediction profiles for objective and constraints.

for each response listed on the ordinate axis. The center value indicates the value of that response based on the current design variable settings. The upper and lower values indicate the limits that response can take within the design space. Note that the results for $/RPM are scaled with respect to the baseline based on the competitive sensitivity of the data. Wing area is depicted in thousand square feet (ft^2), TIT in thousand degree Fahrenheit (°F), and the Ec-range in thousand nautical miles (n miles).

The R-square value predicts how much of the total variation is accounted for by the variables being analyzed. The high values reported for all responses in Fig. 10, R-square values between 99.1 and 99.9%, imply that the variables, as well as the order of the regression polynomials, do account for most of the variation in the responses. However, a major concern is still the prediction accuracy of this equation at off-design points (points not used for regression), an aspect that is not captured by the R-square measure. For this purpose, an additional DoE table is employed, executing the synthesis code 500 times at random points in the design space. These 500 results are compared with the results predicted by the RSEs at those random points. These comparisons for several key responses are presented in Figs. 11–13, in the form of correlation graphs, as an off-design point-performance measure for the RSE. A designer uses these graphs to compare the predictive capability of the RSEs using the 500 random executions of the synthesis code. A perfect correspondence of both data sets would be indicated by a 45-deg line through all data points from the bottom left to the top right corner. This would represent the case of a correlation value of 1. As the graphs indicate, the correlation for each response is quite good, ranging from 0.9965 for the $/RPM to 0.9126 for the approach speed. The graphs also display a prediction ellipse, representing a confidence interval that encloses 95% of the data points in each graph.

Steps 4 and 5: RDS Results

The obtained RSE for the objective function, $/RPM, can now be employed in a Monte Carlo simulation. The purpose of this simulation is to obtain a frequency plot for $/RPM resulting from the intrinsic variability of the economic noise variables. Because of a lack of more precise knowledge about the economic variables, triangular continuous distributions are assumed, which are defined by a minimum, maximum, and a most likely value that may skew the distribution. For example, the distributions for fuel cost and economic range are depicted in Fig. 14. Not shown is the load factor, which has a symmetric triangular distribution with a most likely value of 0.65. To identify the dependence of the objective distribution on the design variables, another face-centered CCD is constructed, this time for the six control variables only (Table 4). A Monte Carlo simulation is executed for each of the cases listed in Table 4. Each case sets the control variables to a fixed value, whereas the noise variables are varied according to their assigned distributions. Each case yields a frequency distribution for $/RPM that can be approximated by a standard gamma distribution that is uniquely defined by its three parameters: location, shape, and scale (Fig. 15). The probability of achieving values below four sample targets, A, B, C, and D, is collected for each distribution and regressed by a second-order polynomial (R-square value of 0.98–0.92). These RSEs for $P(Y \leq T_i)$ allow the designer to functionally relate probability levels back to design variables, and thus, enable the robust

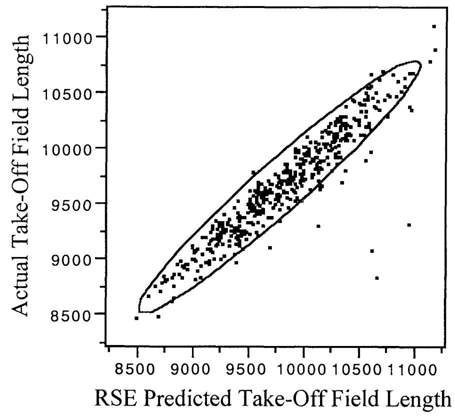

Fig. 13 Correlation of actual with predicted values for takeoff field length.

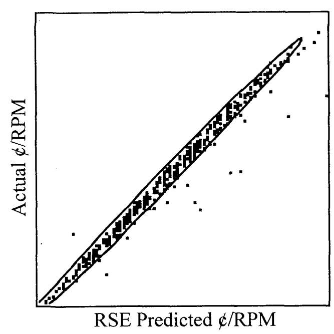

Fig. 11 Correlation of actual with predicted values for $/RPM.

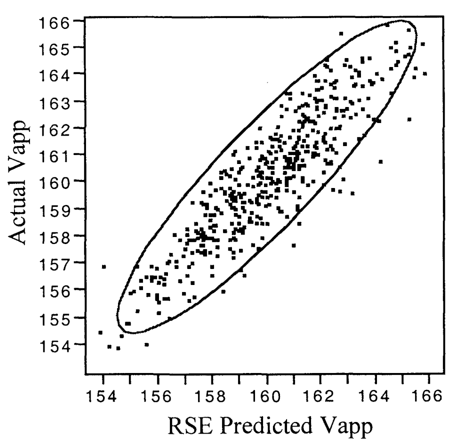

Fig. 12 Correlation of actual with predicted values for approach speed.

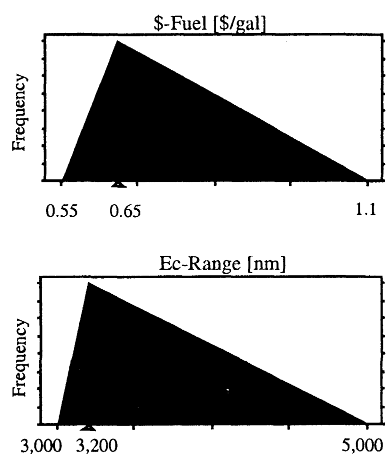

Fig. 14 Distribution for fuel cost and economic range.

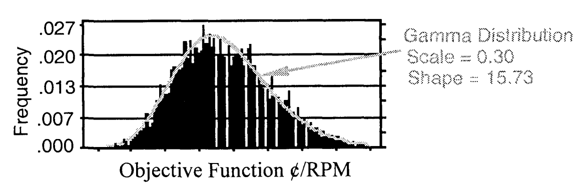

Fig. 15 Objective distribution fit.

Table 4 DoE table and noise variable distributions to obtain objective distribution

	Control Variables						Noise Variables			Response		
Exp.#	T/W-Ratio	S-Wing	x1	y1	TIT	FPR	$-Fuel	Load Factor	Ec-Range	¢/RPM	Shape	Scale
1	0.28	8.5	1.54	0.5	3	3.5					0.3	14.86
2	0.28	8.5	1.54	0.5	3.25	4.5					0.33	13.24
3	0.28	8.5	1.54	0.58	3	4.5					0.34	11.77
4	0.28	8.5	1.54	0.58	3.25	3.5					0.28	15.5
5	0.28	8.5	1.62	0.5	3	4.5					0.35	13.25
6	0.28	8.5	1.62	0.5	3.25	3.5					0.31	15.38
7	0.28	8.5	1.62	0.58	3	3.5					0.31	14.81
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
57	0.3	9	1.58	0.54	3.125	4					0.32	14.02
58	0.3	9	1.58	0.54	3.125	4					0.33	13.85

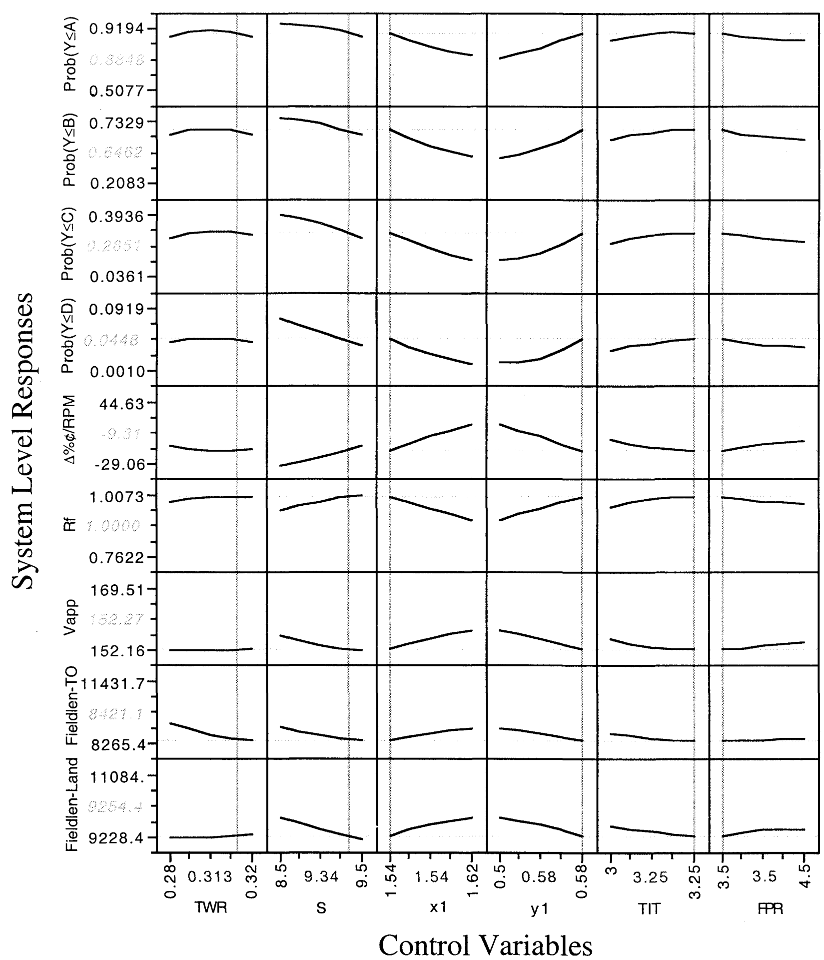

Fig. 16 Prediction profiles for probabilities, objectives, and constraints.

optimization outlined previously. However, this optimization procedure must not allow violation of the constraints.

The robustness optimization for the HSCT completes the implementation of the methodology described in Fig. 3. The RSEs for the probability of achieving objective values below the target values A, B, C, and D, together with the constraints of interests, are displayed in Fig. 16 in the form of prediction profiles. Using these RSEs, an optimization is performed using a standard grid search technique to obtain the settings of the design variables that maximize Prob($Y \leq$ Target A) while satisfying constraints on fuel volume, approach speed, and the takeoff and landing field lengths. The resulting optimal settings are shown in Fig. 16, indicated by the hairline associated with each variable. Once again, for each response, the center value indicates the current value of that response based on the design variable settings. The RDS results in a probability of about 0.88 that target A for $/RPM will be achieved. The associated reduction in probability levels for the more aggressive targets can be noted as well. The computational efficiency of the RSEs allows the relatively unsophisticated grid search to be used,

Table 5 RDS summary

Parameter	Robust solution	Baseline
TWR	0.313	0.300
S	9340 ft^2	9000 ft^2
x1	1.54 semispan	1.58 semispan
y1	0.58 semispan	0.54 semispan
TIT	3250°F	3125°F
FPR	3.5	4.0

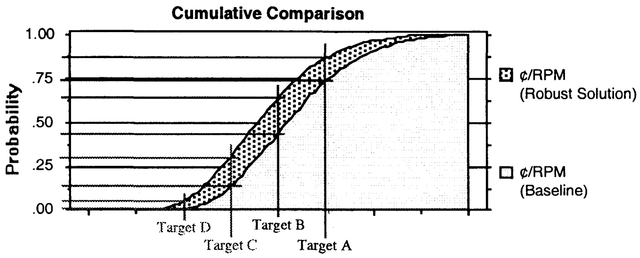

Fig. 17 Distribution comparison between baseline and RDS.

though more sophisticated optimization techniques, such as gradient-based methods, could be employed instead. The resulting robust design solutions are summarized and compared with baseline in Table 5.

Based on the results presented in this table, a Monte Carlo simulation is used one more time to compare the cumulative distribution for $/RPM of this robust design solution to the original one of the baseline HSCT, as displayed in Fig. 17. For all four targets for $/RPM, the robust design solution yields a higher probability of achieving values below the respective targets. Naturally, the probability increases with increasing values for the target. However, for small and very large target values, the difference in probability between the robust design solution and the baseline is very small. The difference increases for values around the means of the distributions. A conclusion from this study is that the difference in probabilities does depend on the target itself. However, it should be pointed out that the baseline and robust solutions are determined for an initial target. Subsequent targets can be overlaid to examine alternative customer imperatives.

As stated in the Introduction, the design activity is concerned with a designer using available information to make intelligent decisions leading to optimal solutions. The information generated by the RDS, encapsulated in Figs. 16 and 17, translates to extremely useful knowledge to designer. Ultimately, a designer wants to know the following: are there solutions within the design space that meet the customer objectives. A deterministic design approach would answer yes or no, and if yes, identify the optimal solution. In contrast, the RDS presented here would answer, yes or no, with associated probability estimate for specified target, and identify the robust solution. This additional information obtained by treating the problem probabilistically is invaluable in the authors' opinion, an is the correct way to approach design problems. In summary, the RDS assists in achieving the goal of answering the question, should the vehicle be built, as opposed to, can the vehicle be built.

Conclusions

Uncertainty is often neglected in aerospace applications of multidisciplinary design, analysis, and optimization algorithms. To account for such uncertainty and investigate its impact on design selection, a design methodology is proposed that is probabilistic in nature and focused on achieving robustness (as opposed to a deterministic optimum or a pure variability minimization). The construction of cumulative distribution functions culminate the execution of the method, and these functions are used by the designer to select the alternatives that maximize the probability of success within a design space specified by a ranged set of control and noise variables. The advantages and differences of this new approach, with respect to other approaches, were described and demonstrated via an application problem involving the system level design of a HSCT. This application demonstrated the ability of the approach to assist a designer in the identification of key design drivers, model important outcomes via the RSM, and use these models in a Monte Carlo simulation to generate the desired probability distributions. Further, the dependency of the objective on the uncertainty parameters was clearly exposed. A robust design solution for the civil transport was obtained and can be used in subsequent stages of design.

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